

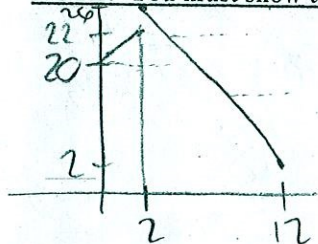
SCORE: 27 / 30 POINTS

1. No calculators / notes / unauthorized paper, electronics or communication allowed
2. Simplify all answers unless stated otherwise
3. Show proper calculus level work to justify your answers

A person's velocity (in meters per second) at time t (in seconds) is given by $v(t) = \begin{cases} 20+t, & 0 \leq t \leq 2 \\ 26-2t, & 2 \leq t \leq 12 \end{cases}$ SCORE: 3 1/2 / 5 PTS

a) Find the exact distance the person travelled from time $t = 0$ seconds to $t = 12$ seconds.

NOTE: You must show the arithmetic expression that you used to get your answer.



$$26 - 2(12) = 26 - 24 = 2$$

$$\left[\frac{1}{2}(20+22)2 + \frac{1}{2}(26+2)10 \right] = \int_0^{12} v(t) dt$$

$$\textcircled{1} 42 + 140$$

182 meters

b) Estimate the distance the person travelled from time $t = 0$ seconds to $t = 12$ seconds using three subintervals and left endpoints.

NOTE: You must show the arithmetic expression that you used to get your answer.

$$\frac{12-0}{3} = 4 = \Delta t$$

$$4 [v(0) + v(4) + v(8)]$$

$$4 (\textcircled{20} + \textcircled{18} + \textcircled{10})$$

$$4(48)$$

$$\frac{3 \cdot 48}{4} = 192$$

192 meters

The graph of function f is shown on the right.

The graph consists of a diagonal line, an arc of a circle, then two additional diagonal lines.

SCORE: 2 1/2 / 4 PTS

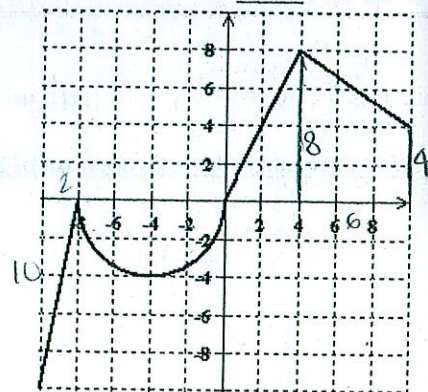
1) Evaluate $\int_{-10}^{10} f(x) dx$.

NOTE: You must show the arithmetic expression that you used to get your answer.

$$-\left[\frac{1}{2}(2)(10) \right] - \left(\frac{1}{2} \cdot \pi \cdot 2^2 \right) + \frac{1}{2}(4)(8) + \frac{1}{2}(8+4)6$$

$$\textcircled{1} -10 - 2\pi + 16 + 36 \textcircled{2}$$

$$\boxed{42 - 2\pi}$$



1) Evaluate $\int_{10}^{-4} f(x) dx = - \int_{-4}^{10} f(x) dx$

$$-\left[-\frac{1}{4}\pi 2^2 + 16 + 36 \right]$$

$$-\left[-\pi + 52 \right] = \boxed{\pi - 52}$$

Using the limit definition of the definite integral, and right endpoints, find $\int_{-4}^{-1} (2x^2 + 8x) dx$.

SCORE: 10 / 10 PTS

NOTE: Solutions using any other method will earn 0 points.

$$x_i = a + i\Delta x$$

$$\Delta x = \frac{-1 - (-4)}{n} = \frac{3}{n}$$

$$\left(\frac{3i}{n} - 4\right)\left(\frac{3i}{n} - 4\right) = \frac{9i^2}{n^2} - \frac{24i}{n} + 16$$

$$(n^2 + n)(2n + 1) = 2n^3 + n^2 + 2n^2 + n = 2n^3 + 3n^2 + n = n^3\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n (2x_i^2 + 8x_i) \Delta x$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[2\left(-4 + \frac{3i}{n}\right)^2 + 8\left(-4 + \frac{3i}{n}\right) \right] \frac{3}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[2\left(16 - \frac{12i}{n} - \frac{12i}{n} + \frac{9i^2}{n^2}\right) + \left(-32 + \frac{24i}{n}\right) \right] \frac{3}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[32 - \frac{48i}{n} + \frac{18i^2}{n^2} - 32 + \frac{24i}{n} \right] \frac{3}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(-\frac{24i}{n} + \frac{18i^2}{n^2} \right) \frac{3}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n -\frac{72i}{n^2} + \frac{54i^2}{n^3}$$

$$\lim_{n \rightarrow \infty} -\frac{72}{n^2} \sum_{i=1}^n i + \lim_{n \rightarrow \infty} \frac{54}{n^3} \sum_{i=1}^n i^2$$

$$\lim_{n \rightarrow \infty} -\frac{72}{n^2} \cdot \frac{n(n+1)}{2} + \lim_{n \rightarrow \infty} \frac{54}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = -36 + 18 = -18$$

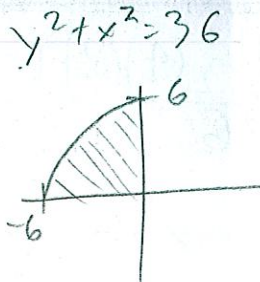
$$\lim_{n \rightarrow \infty} \frac{54}{n^3} \cdot \frac{n^3\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)}{6} = 9 \cdot \left(2 + \frac{3}{n} + \frac{1}{n^2}\right) = 18$$

+ (1) $\lim_{n \rightarrow \infty}$

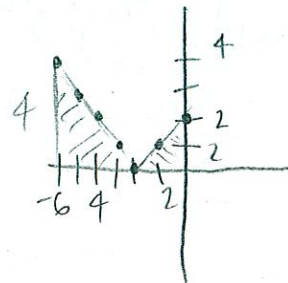
Evaluate $\int_{-6}^0 (2\sqrt{36-x^2} - |x+2|) dx$ using the properties of definite integrals and interpreting in terms of area. SCORE: 5 / 5 PTS

NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

$$2 \int_{-6}^0 \sqrt{36-x^2} dx - \int_{-6}^0 |x+2| dx = 2(9\pi) - 10 = 18\pi - 10$$



$$\frac{1}{4} \pi 6^2 = \frac{1}{4} \cdot 36\pi = 9\pi$$



$$\frac{1}{2}(4)(4) + \frac{1}{2}(2)(2) = 8 + 2 = 10$$